GRAPHICAL METHOD FOR PROFILING THE TOOLS WHICH GENERATE INTERNAL SURFACES BY ROLLING

Silviu BERBINSCHI¹, Virgil Gabriel TEODOR¹, Gabriel R. FRumușANU¹ and Nicolae OANCEA¹

ABSTRACT: In this paper, we present a graphical method, developed in CATIA environment, based on the method of substituting circles family and dedicated to profile pinion cutters for generating internal surfaces. Hereby, the family of substituting circles associated to the worked piece (the profile to be generated) is determined at the same time to the curve transposed, in the rolling motion, to the tool centrode. After finding the position of the contact points between the generated profile and the family circles, we can determine, in the transposed family, the locus of the points from the reciprocal enwrapping profile – the pinion cutter profile. The paper also includes two method applications, for generating a hole with square transversal section, and a pump stator with trapezoidal profile. Comparisons between the profiling results when using the graphical method, versus an analytical method have also been realized. The results are proving the coincidence between the two profiles determined as above, on one hand, and both rapidity and precision of the graphical method, on the other hand.

KEY WORDS: graphical method, CATIA, profiled holes, pinion cutter.

1 INTRODUCTION

The profiled hole definition, accepted here, is: cylindrical surface having the transversal section formed by an ordinate whirl of profiles. This type of surfaces have technical application in detachable clutches construction (holes with square or hexagonal sections, slots with parallel flanks or triangular). They can be also met at the stators of special pumps (2z/z+1 pumps, gear rims with interior teeth). The generation of such surfaces can be done by milling with machine templates, in the case of a square hole (Berbinschi et al, 2014), by broaching with profile shaped broaches – if the required broach dimension is satisfactory from economical point of view, or by sloting with pinion cutters (Oancea, 2004).

Profiling the pinion cutter used to generate an ordinate whirl of profiles, which constitutes a hole generatrix, can be realized by appealing to fundamental theorems of the enwrapped surfaces by rolling – GOHMAN (Litvin, 1989), (Radzevich, 2007) or to complementary analytical methods, such as: “the minimum distance method”, “the family of substituting circles method” (Oancea, 2004), or “the plain trajectories method” (Teodor, 2010).

In the present paper we suggest a solution for this type of problem, based on the family of substituting circles method, and developed in a graphical environment – CATIA. This problematic has been already approached for the cases of generating with a rack-tool or with a pinion cutter (but for exterior whirls of profiles) (Ivanov et al., 1998), (Berbinschi et al., 2012, 2013). The graphical method applies the facilities of CATIA graphical environment, which enables to give a rapid and rigorous solution for profiling the pinion cutter used to generate interior surfaces. The qualities and the accuracy of the new-developed method are assessed by comparison to the known analytical methods.

2 THE “FAMILY OF SUBSTITUTING CIRCLES” METHOD

We further present the family of substituting circles method, applied in the particular case of enwrapping between profiles associated to a couple of interior rolling centrodes (Fig. 1). The following reference systems are considered:

- \( xy \) – meaning a global system, attached to \( C_1 \) centrode axis;
- \( x_0y_0 \) – global system, solidary to the rotation axis of \( C_2 \) centrode;
- \( XY \) – relative system, attached to \( C_2 \) profile to be generated and also to \( C_1 \);
- \( \xi\eta \) – relative system, solidary to \( C_2 \) centrode of the pinion cutter.

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According to “family of substituting circles” method, the envelop of $C_\Sigma$ profile, in the rolling motion between the two centrodes, is defined by the transposed of the circles family to $C_\Sigma$ centrode, associated to the tool, in the relative motion:

$$\xi = \omega_2 (\varphi_2 \omega_1^T(\varphi_1) \cdot X - A),$$

with

$$A = \begin{bmatrix} -a \\ 0 \end{bmatrix},$$

where $X$ matrix means the circles family (1), with $r_i$ and $\theta_i$ given by conditions (5), see also (4).

3 GRAPHICAL METHOD

We further present a graphical method, developed in CATIA environment for profiling the pinion cutter type of tool, used to generate an ordinate whirl of profiles, defining an ensemble of interior surfaces (for example polygonal holes, interior slots, interior gear rims with involute profile teeth, pump stators).

Fig. 2 illustrates the principle of determining the substituting circles family for $C_\Sigma$ profile, based on the following succession of steps:

- For the section delimited by the characteristic points $A$ and $B$, the normal to profile are drawn in these points. Their intersection with the circle of $R_{ip}$ radius determines the points $a$ and $b$ (see Fig. 2), corresponding to an angle at center denoted by $\delta$.
- The angle $\delta$ is divided in $n$ equal parts, or the coordinates of $O_{i,p}$ points are arbitrary chosen on the circle of $R_{ip}$ radius, as well.

![Figure 2. The substituting circles family determined in CATIA environment](image-url)
The circles tangent to $C_\Sigma$ profile, and having the centers in the $O_{i,p}$ points, are drawn (PROJECT and CIRCLE commands).

The $\theta_{i,p}$ angles, formed between $O_iO_{i,p}$ radius and the normal in $O_{i,p}$ to $C_\Sigma$ profile are measured.

The co-ordinates of $M_i$ points – the tangency point between the circle $O_{i,p}$ and $C_\Sigma$ profile are determined (PROJECT command).

These graphical procedures are repeated for all the circles from $C_\Sigma$ profile substituting family, and then the circles family is transposed on the tool centrode, as it follows (see also Fig. 3).

The $O_{i,s}$ points, owning to the circle of $R_{rs}$ radius, associated to the pinion cutter, are determined. They are corresponding to $O_{i,p}$ points from $C_i$ centrode, in the rolling motion, see also relation (3). The angle between two successive points, $O_{i,s}$ and $O_{i+1,s}$ is $i\cdot\delta/n$.

The circles of $r_i$ radius and having their centers in $O_{i,s}$ points are drawn (CIRCLE command).

The straight line $N_i$, forming the angle $\theta_{i,p}$ with $O_2O_{i,s}$ direction, is drawn – see Fig. 2; at its intersection with the circle of $r_i$ radius we have the point $M_{i,s}$. The totality of $M_{i,s}$ points, determined as above, are giving the pinion cutter profile, $S$, reciprocal enwrapped to $C_\Sigma$ profile, in the rolling process between the two centrodes $C_1$ and $C_2$.

4.1 The pinion cutter for generating a square hole

Fig. 4 illustrates the graphical method application in order to solve the tool-profiling problem when generating a square hole with a pinion cutter. The needed reference systems and the geometrical parameters, which have to be considered are here presented.

The radius $R_{rp}$ of the centrode associated to the generated profile has to satisfy the condition that the normal, considered in any of the profile points, intersects the circle of $R_{rp}$ radius. Due to the symmetry, it is sufficient to solve the problem for the $AB$ segment (Fig. 4).

In the case of the analytical solution, the profile $C_\Sigma$, $[AB]$ segment, is given through the equations:

$$X = -L;$$

$$Y = u;$$

with $u$ variable respecting the restrictions $0 \leq u \leq L$. The condition to find the family of substituting circles leads to the following relations (see also Fig. 4):

$$\sin \phi_i = \frac{u}{R_{rp}}; \quad \theta_i = 0;$$

$$r_i = -L + \sqrt{R_{rp}^2 - u^2}; \quad R_{rp} = \sqrt{2} \cdot L,$$

which, when $u$ remains inside the limits above specified, determines the substituting circles family.
According to the algorithm presented in the previous section, the family of circles is also found with the help of CATIA environment graphical facilities (see Fig. 5), when \( L = 20 \) mm, and \( R_{rp}/R_{rs} = 4/3 \).

The pinion cutter profile, as obtained by both analytical and graphical methods, is represented in Fig. 6. At the same time, in Table 1 are comparatively exemplified the co-ordinates of some among the points determining the tool profile, as they resulted from each of the two methods application.

<table>
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<tr>
<th>Crt. no.</th>
<th>Analytical method</th>
<th>Graphical method</th>
</tr>
</thead>
<tbody>
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<td>( \xi ) [mm]</td>
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<tr>
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As it can be noticed, by analyzing the results in graphical form, the profiles determined by using the two mentioned methods are coincident, even if between the co-ordinates from Table 1 there are some differences. They are due to the fact that, excepting the extremities, we are not talking about exactly the same points along the profile – because of its specific, each of the methods gives the same profile, but through different series of points.

### 4.2 The pinion cutter for generating a pump stator with trapezoidal profile

The pumps with stator having rectilinear-profile teeth are used for operating viscous fluids in drugs or cosmetics industry as well as in factories producing building materials. Rectilinear segments and arcs of circle compose the profile of such a stator tooth (Fig. 7).

Thereby, the stator tooth profile from Fig. 7, referred to \( XY \) system is formed by the following simple curves:

- The arc of circle \( AB \) of \( r_1 \) radius, having the equations
  \[
  \begin{align*}
  X &= -R_{ch} - r_1 \cos \theta_1; \\
  Y &= -r_1 \sin \theta_1,
  \end{align*}
  \]
  with \( \theta_1 \) variable parameter.
The rectilinear segment $BC$, of equations
\[
\begin{align*}
X &= -R_{o1} - r_1 \sin \beta + u \cos \beta; \\
Y &= -r_1 \cos \beta - u \sin \beta,
\end{align*}
\] (11)

with $u$ variable parameter.

The arc of circle $CD$, of equations
\[
\begin{align*}
X &= -R_{o2} \cos \left( \frac{\delta}{2} \right) - r_2 \cos \left( \theta_1 + \frac{\delta}{2} \right); \\
Y &= -R_{o2} \sin \left( \frac{\delta}{2} \right) + r_2 \sin \left( \theta_1 + \frac{\delta}{2} \right),
\end{align*}
\] (12)

with $\theta_2$ independent variable.

The limits of variation for variable parameters from equations (10) – (12) can be established according to constructive reasons, as it follows:

- For the angular parameters, we have
  \[
  \begin{align*}
  \theta_{1_{\text{min}}} &= 0; \\
  \theta_{1_{\text{max}}} &= \pi - \beta; \\
  \theta_{2_{\text{min}}} &= 0; \\
  \theta_{2_{\text{max}}} &= \frac{\pi}{2} - \beta.
  \end{align*}
  \] (13)

- For finding the $u$ parameter limits, we firstly determine, with equations (10) and (12) associated to conditions (13) and (14), the co-ordinates of points $B$ and $C$,
  \[
  \begin{align*}
  X_b &= -R_{o1} - r_1 \sin \beta; \\
  Y_b &= -r_1 \cos \beta; \\
  X_c &= -R_{o2} \cos \left( \frac{\delta}{2} \right) - r_2 \sin \left( \beta - \frac{\delta}{2} \right); \\
  Y_c &= -R_{o2} \sin \left( \frac{\delta}{2} \right) + r_2 \cos \left( \beta - \frac{\delta}{2} \right).
  \end{align*}
  \] (15)

Then, we can find
\[
\begin{align*}
u_{\text{min}} &= 0; \\
u_{\text{max}} &= \sqrt{(X_b - X_c)^2 + (Y_b - Y_c)^2}.
\end{align*}
\] (17)

In Fig. 8 there are presented the two rolling centroids, $C_1$ and $C_2$, having the radii $R_{rst}$ and $R_{sc}$, respectively; they must obey the rolling condition:
\[
R_{rst} \cdot \varphi_1 = R_{sc} \cdot \varphi_2.
\] (18)

The following reference systems are defined:

- $xy$ and $x_0y_0$, as absolute systems;
- $XY$ – mobile, attached to the stator profile;
- $\tilde{xy}$ – mobile, attached to the pinion cutter.

The motion of $XY$ system relative to $\tilde{xy}$ system, described by (6), enables us to find, after calculus, the family of $\Sigma$ profiles in the tool reference system. From this point on, the application of one among the already known methods to determine this family’ envelop leads to the solution meaning the pinion cutter profile.

However, if we intend to solve the problem by using the graphical method, then a substituting circles family is firstly associated to the frontal profile of the pump stator (Fig. 9). According to the suggested profiling algorithm, the transposed, the transposed of this family on the tool centerde, of $R_{rst}$ radius is then determined (Fig. 10). The ensemble of $C_i$ circles tangency points with the three curve segments – $AB$, $BC$ and $CD$, along the rolling motion, gives the pinion cutter profile.

The Fig. 11 presents the tool profile obtained for the stator tooth profile characterized by $r_1 = r_2 = 3$ mm, $\delta/2 = 30^\circ$, $R_{O1} = 55$ mm and $R_{O2} = 45$ mm.
5 CONCLUSION

The graphical method suggested in this paper turns to profit the facilities of CATIA graphical environment. The solution offered by the graphical method is rigorous (this being proven by comparison with analytical methods), rapid, and, at the same time, intuitive. The method is also appropriate for studying complex interior surfaces and has the important advantage of diminishing the risk of giving a wrong interpretation to numerical data regarding a pinion cutter profiling.

6 ACKNOWLEDGEMENTS

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7 REFERENCES


In Table 2 there are also exemplified the co-ordinates of some among the points from the tool profile, found by applying the graphical method.

<table>
<thead>
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