SYSTEMIC APPROACH OF BEARING ARRANGEMENT LIFE

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ABSTRACT: The purpose of the present paper is to briefly illustrate a new systemic approach for calculating bearing loads and subsequently, the compressive forces that load the balls of the bearings which support a shaft. The proposed method is grounded on the following theory: when the static equilibrium of the shaft is reached, the loads and moments transmitted from the shaft to the bearings must be reacted by the loads and moments arisen due to the elastic deformations of the rolling elements. The method is applied to bearing life calculation for a mechanical application in order to prove the suitability of the proposed method for double-row bearings or paired bearings.

KEY WORDS: double-row bearings, bearing arrangement life, systemic approach.

1 INTRODUCTION

Bearing life calculation is a major aspect of the rolling bearing industry and an accurate prediction of the bearing life is the goal of every respectable company from this industry. Due to this fact intensive research has been conducted in order to find reliable mathematical models and methods that accurately predict the bearing life.

Bearing life can be calculated using ISO 281:2007 or ISO/TS 16281:2008. ISO 281:2007 is used to calculate the basic rating life and it also stipulates approaches for calculating the modified rating life in which more aspects such as fatigue load or lubrication condition are taken into consideration. Calculating the bearing life according to ISO/TS 16281:2008 (ISO/TS 16281, 2008) leads to more realistic values due to the fact that the bearing clearance, tilting angle, and moment load are considered.

However, regardless of the method applied (e.g. following ISO 281 or ISO 16281), the prediction accuracy of bearing rating life largely depends on the values of the loads and moments acting on the bearing, used as input data in the calculation. Considering the shaft on which the bearings are mounted a statically loaded beam, with the external loads and bending moments originating from the machine elements mounted on the shaft, is a current practice. The supports of this beam are, unquestionably, the bearings mounted on the shaft, in which the values of loads and tilting moments acting on the bearings are equal to the values of the calculated support reactions and reaction moments.

The problem of the support reactions calculation is elementary when the shaft rests on two bearings—the beam is statically determinate—but becomes rather complicated when the shaft is supported by more than two bearings, since the beam becomes statically indeterminate. In this case, the beam deflections have to be considered, thus, more shaft sections with different moment of inertia increase the problem difficulty.

In this paper a systemic approach for bearing arrangement life calculation is proposed and exemplified on an arrangement consisting in a shaft supported by a double-row angular ball bearing and by a deep groove ball bearing. The early work regarding the basis of the approach presented in this paper is included in (Tudose, 2013).

2 PROPOSED BALL BEARING LIFE MODEL

Unlike the ISO 281 procedure (ISO 281, 2007), according to ISO 16281 (ISO/TS 16281, 2008) the values of the compressive loads acting on each ball of the bearing are crucial in ball bearing lives calculation. As one can see in the following paragraphs the ball compressive loads are functions of the ball deformations and these are in turn functions of the relative displacements of the inner ring with respect to the outer ring. Consequently, a first goal to be achieved is to find the linear and angular displacements of the bearing reference points.

The concept lying behind the proposed method is based on a system approach: when the static equilibrium of the shaft is reached, the loads and moments (vector \( \mathbf{FM}(\Delta \Theta) \)) transmitted from the shaft to the bearings must be reacted by the loads and moments (vector \( \mathbf{fm}(\Delta \Theta) \)) arisen due to the elastic deformations of the rolling elements:

\[
\mathbf{FM}(\Delta \Theta) + \mathbf{fm}(\Delta \Theta) = 0
\]
The former are obtained using the slope-deflection method (very likely never used before in this context). One of the first comprehensive descriptions of the slope deflection method can be found in (Matheson, 1960). The latter result from a certain load-deformation model of rolling elements of the bearing. The unknowns of the equation (1) are the displacements of the shaft support reference points (nodes \( k_1 \) and \( k_n \) Fig. 3) given by:

\[
\Delta \Theta = \left( \delta_i \delta_{i+1} \theta_i \theta_{i+1} \right)
\]

which are determined first by solving the equations of equilibrium. \( \Delta \Theta \) contains those five displacements per bearing node and because the shaft is considered perfectly rigid in axial direction, the axial displacements of the two considered nodes are equal and consequently, the corresponding value is considered only once. Once these displacements are found, the unknown forces are obtained through force-displacement relations.

### 2.1 Loads and moments transmitted from the shaft to the bearings

A shaft and its \( n \) discretization nodes are given in Fig. 1. The nodes could represent supports, points where external loads and moments are applied, points at which the shaft changes its section, etc. Each segment \( i,i+1 \) is characterized by length \( L_i \) and moment of inertia \( I_i \).

\[ F_{i+1} = (F_i, F_{i+1})^T \]

The slope-deflection equations (Megson, 2005) establish force-displacement relationships for the segment \( i,i+1 \):

\[ F_{i+1} = 6 \cdot c_i \delta_i - 2c_i \delta_{i+1} + b_i \theta_i + b_{i+1} \theta_{i+1} + F_{i+1}^f \]

\[ F_{i+1} = -6 \cdot c_i \delta_i - 2c_i \delta_{i+1} + b_i \theta_i + b_{i+1} \theta_{i+1} + F_{i+1}^s \]

\[ M_{i+1} = -2 \cdot (3b_i \delta_i - 3b_i \delta_{i+1} + a_i \theta_i + a_{i+1} \theta_{i+1}) + M_{i+1}^f \]

in which

\[ a_i = E_i \frac{L_i}{L_i}, b_i = \frac{L_i}{L_i}, i = 1, \ldots, n - 1 \]

\( E_i \) is the modulus of elasticity of the shaft material, \( M_{i+1}^f = -M_i \) and \( M_{i+1}^f = -M_{i+1} \) are the fixed-end shear moments, and \( F_{i+1}^f = -F_i \) and \( F_{i+1}^f = -F_{i+1} \) are the fixed-end shear forces.

### Figure 2. Slope and deflection of a shaft element

The elements of the following vector are obtained by summing Eqn. (4)-(5) and then Eqn. (6)-(7) in each node:

\[ RrM = (R_r, R_s, M, rM_1, \ldots, rM_n) \]

Using the denotation \( \delta \Theta = (\delta_1, \delta_2, \ldots, \delta_n, \Theta_1, \Theta_2, \ldots, \Theta_n)^T \), one obtains the succeeding matrix equation:

\[ RrM = AG \cdot \delta \Theta - FM \]

where \( AG \) is the stiffness matrix. If \( i \) is a free node, then \( RrM_i = \delta = 0 \) and \( RrM_{i+1} = rM_i = 0 \). If \( i \) is a support, then \( RrM_i \) and \( RrM_{i+1} \) could be non-zero and represent the reaction and the reaction moment of the support against the shaft. Presuming that the following \( q \) nodes, \( q_1, q_2, \ldots, q_q \) are the shaft supports (in ascending order) let’s extract the vector \( \delta \Theta^{(p)} = (\delta_{q_1}, \delta_{q_2}, \ldots, \delta_{q_q})^T \) from the vector \( \delta \Theta \) and let’s symbolize the remaining vector with \( \delta \Theta^{(o)} \). We introduce two new matrices \( AG^{(p)} \) and \( AG^{(o)} \), where \( AG^{(i)} \) results from \( AG \) by erasing the columns \( q_1, q_2, \ldots, q_q \) and \( AG^{(p)} \) will be formed by the erased columns. Eqn. (9) becomes:

\[ RrM = AG^{(p)} \cdot \delta \Theta^{(p)} + AG^{(o)} \cdot \delta \Theta^{(o)} - FM \]

The system is separated into two systems corresponding to the free and constrained nodes:

\[ RrM_{\text{free}} = AG^{(p)} \cdot \delta \Theta^{(p)} + AG^{(o)} \cdot \delta \Theta^{(o)} - FM_{\text{free}} \]

\[ RrM_{\text{cstr}} = AG^{(p)} \cdot \delta \Theta^{(p)} + AG^{(o)} \cdot \delta \Theta^{(o)} - FM_{\text{cstr}} \]

\( RrM_{\text{free}} \) corresponds to the free nodes, and therefore this vector is null. The matrices \( AG^{(p)} \) and \( AG^{(o)} \) are deduced from \( AG^{(p)} \) and \( AG^{(o)} \), respectively, by erasing the rows \( q_1, q_2, \ldots, q_q, n+q_1, n+q_2, \ldots, n+q_q \). The matrices \( AG^{(p)}_{\text{cstr}} \) and \( AG^{(o)}_{\text{cstr}} \) are
formed by the erased rows. \( FM_{\text{free}} \) is obtained from \( FM \) by erasing the same rows, while \( FM_{\text{str}} \) is formed by the erased rows. Expressing the elements of the vector \( \delta \Theta^{p} \) as a linear combination of the elements of the vector \( \delta \Theta^{q} \) from Eqn. (12) and substituting them in Eqn. (13), it yields:

\[
RrM_{\text{str}} = A \cdot \delta \Theta^{q} + B
\]  

where

\[
A = AG_{\text{str}}^{(q)} - AG^{(i)} \cdot AG_{\text{free}}^{(p)}
\]

\[
B = AG^{(i)} \cdot FM_{\text{free}} - FM_{\text{str}}
\]

\[
AG^{(i)} = AG_{\text{str}}^{(i)} \cdot \left( AG_{\text{free}}^{(i)} \right)^{-1}
\]  

\[2.2 \text{ Loads and moments transmitted to the shaft due to bearing elastic deformations}\]

The used model is relatively similar to that presented in (De Mul, 1989), yet with certain differences necessary to handle the inherent radial and axial movements of bearing rings due to fits, preload and thermal expansions of bearing parts, housing and shaft. Since our interest is focused on medium to large ball bearing arrangements the following frequently used assumptions were introduced: (1) only ball elastic deformations are included, other deformations being neglected; (2) centrifugal forces acting on the balls, loads generated by interaction from the cage, and frictional forces inside the bearings are neglected; (3) spring constants of the balls are equal and constant with temperature.

Using Eqn. (14) the corresponding equations can be written for both the vertical and horizontal planes in which the shaft is loaded. Assembling these equations, considering two bearings \((n=2)\) and the sign of the corresponding reactions and reaction moments, rearranging the rows in a convenient manner, and taking into account the external axial force \( F_{\text{ax}} \), the vector \( FM \) from Eqn. (1) can be found:

\[
FM = A \cdot \Delta \Theta + B + FA
\]  

where \( FA=(F_{x} 0 0 0 0 0 0 0)_{T} \) and the displacement vector \( \Delta \Theta \) is given by Eqn. (2).
Because the displacements are usually small, the
displacement vectors $u$ and $\delta \Theta$ can be related using
$$u = T_{nx2xy}(O, \phi) \cdot \delta \Theta,$$
where the transformation matrix is given by the equation:
$$T_{nx2xy}(O, \phi) = \begin{pmatrix}
  0 & \cos \phi & \sin \phi & -x_{0b} \sin \phi & -x_{0b} \cos \phi \\
  1 & 0 & 0 & r_{ab} \sin \phi & -r_{ab} \cos \phi
\end{pmatrix} \tag{19}
$$

The ball force vector $Q$ is transformed to an
equivalent vector $fm$ at the inner ring reference point
by using $fm = T_{nx2xy}(O, \phi) \cdot Q$, where $T_{nx2xy}(O, \phi)$ is the
transpose of the matrix given by Eqn. (19). The load
vector is
$$Q = Q \cdot T_{nx2xy}(O, \phi),$$
where $Q$ is the compressive load acting on the ball, $T_{nx2xy}(O, \phi) = (-\cos \alpha - \sin \alpha)^T$, and
$\alpha$ is the contact (Fig. 6). Based on classical Hertzian
theory $Q = c_p \cdot \delta^{3/2}$, where the $c_p$ is the spring constant and
$\delta$ is the total elastic deformation of the ball.
Therefore,
$$fm = c_p \cdot \delta^{3/2} \cdot T_{nx2xy}(O, \phi, \alpha) \tag{20}$$

where $T_{nx2xy}(O, \phi, \alpha) = T_{nx2xy}(O_0, \phi) \cdot T_{nx2xy}(O_0, \phi)$. In Fig. 6, $r_i$ and $r_e$ are the radii of the inner ring
and outer ring raceway profiles in the axial plane, respectively, $D_w$ is the ball diameter, and $D_e$ is the outer ring groove center. As $O_i$ the point $O_i$ is stored
by means of the subsequent vector $O_e = (r_{0e}, x_{0e})^T$. The contact angle and the deformation of the ball are given by the actual position of $O_i$ and:
$$\alpha(O_i, O_e) = \tan \frac{x_{0e} - x_{0e}}{r_{0e} - r_{0e}} \tag{21}$$
$$\delta(O_i, O_e) = \max \left( 0, -\frac{r_{0e} - r_{0e}}{\cos \alpha(O_i, O_e)} + D_x - r_i - r_e \right) \tag{22}
$$

Keeping in mind that the sum $D_w - r_i - r_e$ is practically constant with temperature variation, the
status of the bearing is uniquely described by the vectors $O_i$ and $O_e$. This approach is extremely helpful
also because one can discern between the status of the bearing “before the external loading” (e.g.
preload/clearance after mounting) and its status “after the external loading”. If the initial status of the
bearing is described by the vectors $O_i$ and $O_e$, the final status (after the external loading of bearing) is
depicted by the vectors $O_i + u_i$ and $O_e = O_e$.
Consequently, knowing the initial status of the bearing before loading and using the equations
presented above for every ball $j$ (of those $Z$ balls), the
loading from the balls on the inner ring will be:
$$fm = c_p \cdot \sum_{j=1}^{Z} \delta^{3/2} \cdot (O_i + u_j, \phi_j, \alpha_j) \tag{23}$$

It is obvious that for a given initial status of the bearing
the vector $fm$ from (23) is a function of only the
displacement vector $\delta \Theta$. Note that in our
approach the initial status of the bearing before
loading always encompasses the elastic deformation
of the bearing rings due to fits and the thermic
expansions of the bearing rings and shaft (both radial
and axial).
3 APPLICATION PRESENTATION

The application from this paper focuses on double-row bearings and how to deal with them within the proposed approach. The solid shaft (Fig. 7a) is supported at the left side (node 5) by a double-row angular ball bearing (basic designation 305608) as locating bearing — and at the right side (node 11) by a deep groove ball bearing (basic designation 61828) as non-locating bearing. Since there is no risk of confusion, we will refer to them as bearing 1 (the double-row bearing) and bearing 2, respectively. The axial internal clearance of the double-row bearing is 80 μm and the radial internal clearance of the deep groove ball bearing is 40 μm. The speed of the shaft is 1000 rpm. The properties of the materials from this application are given in Table 1.

The shaft geometry is given in Table 2 and its loading in two perpendicular planes is detailed in Fig. 7b (the horizontal plane) and Fig. 7c (the vertical plane) and the values are indicated in Table 3. The values of the basic and static radial dynamic load ratings of the bearings are $C_{r1}=130$ kN and $C_{d1}=191$ kN (bearing 1) and $C_{r2}=37$ kN and $C_{d2}=42$ kN (bearing 2). Regarding the bearing seats, it was considered that the outer surfaces of shaft journals are ground and the roughness is $R_{a1}=0.8$ μm. The same roughness (i.e. $R_{a1}=0.8$ μm) was accepted for the housing bore surfaces. The tolerance of the shaft journal diameter is m6 and the one of the housing bore diameter is H7 (same for both bearings). The temperature distributions are given in Fig. 4. The equivalent raceways diameters of the bearing rings were $d_{or1}=174.596$ mm and $D_{or1}=200.894$ mm (bearing 1), and $d_{or2}=149.563$ mm and $D_{or2}=165.437$ mm (bearing 2). For the spring constants of the ball raceways contacts, the values $c_{r1}=433.461$ kN·mm$^{-3/2}$ (bearing 1) and $c_{r2}=340.836$ kN·mm$^{-3/2}$ (bearing 2) were used. Details regarding the mounting dimensions and tolerances can be found in Table 4.

4 BEARING LIFE CALCULATION

The reference point of the bearing 1 is taken in the bearing center (Fig. 8) and as for the single-row angular ball bearings two systems of coordinates (Cartesian and cylindrical) were set in this point. For bearing 2 we proceeded similarly. Based on the bearing ring expansions/contractions (calculated with the most probable values of the expansions/contractions) the coordinates (in own systems) of the groove centers of inner and outer ring for both bearings were computed. We considered the double-row angular ball bearing as one single deformable support of the shaft and to take each ball row of the bearing as an independent shaft support (even if they are looked as deformable) will lead to wrong results (higher balls loads and consequently, lower bearing lives). We suggest the same approach for single-row angular contact ball bearings paired side by side or with short spacers. If the used spacers are long enough, each bearing must be considered an independent support.

An equation similar to equation (1) was aggregated, the solution being presented in Table 5. Using this solution the values of the shaft reaction, moment reaction, deflection and rotation in each support are calculated. When using the same data for a calculation with rigid supports large differences can be noted. In Table 6 the resulted bearing loads are shown and the distribution of the compressive loads among the balls of both bearings can be observed in the Fig. 9.

![Figure 7. Double-row bearing Cartesian and cylindrical coordinate systems. Inner ring loading and displacements](image)

Table 7 contains all necessary data concerning bearing lubrication and, assuming a life modification factor for reliability $a_{1}=1$, both basic rating life $L_{10h}$ and modified rating life $L_{10mth}$ were calculated according to ISO 16281 and ISO 281, respectively. In the case of ISO 281, the bearing loads were obtained from the equilibrium of the shaft rested on rigid supports. We chose to proceed in this manner because the method encompassed in this standard cannot take into account the values of the tilting moments.
Figure 8. Shaft supported by a double-row angular ball bearing and a single-row deep groove ball bearing:
a) sub-assembly axial section; b) shaft loading in the horizontal plane; c) shaft loading in the vertical plane

Table 1. Mechanical and thermal properties of the used materials

<table>
<thead>
<tr>
<th>Property</th>
<th>Shaft</th>
<th>Housing</th>
<th>Bearing balls and rings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus of elasticity, Gpa</td>
<td>207</td>
<td>98</td>
<td>208</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.3</td>
<td>0.211</td>
<td>0.3</td>
</tr>
<tr>
<td>Coefficient of linear thermal expansion, 10^{-5}/°C</td>
<td>1.28</td>
<td>1.04</td>
<td>1.15</td>
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</table>

Table 2. Shaft geometry

<table>
<thead>
<tr>
<th>Segment</th>
<th>1-2</th>
<th>2-3</th>
<th>3-4</th>
<th>4-5</th>
<th>5-6</th>
<th>6-7</th>
<th>7-8</th>
<th>8-9</th>
<th>9-10</th>
<th>10-11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length, mm</td>
<td>125</td>
<td>39</td>
<td>35</td>
<td>26</td>
<td>30</td>
<td>62</td>
<td>40</td>
<td>15</td>
<td>285</td>
<td>9</td>
</tr>
<tr>
<td>Outer diameter, mm</td>
<td>140</td>
<td>150</td>
<td>155</td>
<td>160</td>
<td>160</td>
<td>162</td>
<td>162</td>
<td>175</td>
<td>150</td>
<td>140</td>
</tr>
<tr>
<td>Inner diameter, mm</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3. Shaft loading

<table>
<thead>
<tr>
<th>Plane</th>
<th>Loading</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>Horizontal XOZ</td>
<td>Belt load, kN</td>
<td>-12.000</td>
</tr>
<tr>
<td></td>
<td>Helical pinion radial force, kN</td>
<td>3.083</td>
</tr>
<tr>
<td></td>
<td>Helical pinion axial force, kN</td>
<td>-2.270</td>
</tr>
<tr>
<td></td>
<td>Moment of the helical pinion axial force, kN \cdot mm</td>
<td>-281.460</td>
</tr>
<tr>
<td>Vertical XOY</td>
<td>Helical pinion tangential force, kN</td>
<td>-8.471</td>
</tr>
</tbody>
</table>

Table 4. Mounting dimensions and tolerances

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Bearing 1</th>
<th>Bearing 2</th>
</tr>
</thead>
</table>
Bearing bore nominal diameter, mm | 160 | 140
---|---|---
Upper limit deviation of the bearing bore diameter, mm | 0 | 0
Lower limit deviation of the bearing bore diameter, mm | -0.018 | -0.025
Bearing outer nominal diameter, mm | 215 | 175
Upper limit deviation of the bearing outer diameter, mm | 0 | 0
Lower limit deviation of the bearing outer diameter, mm | -0.020 | -0.025

| Bearing width (nominal), mm | 56 | 18
| Shaft nominal outer diameter, mm | 160 | 140
Upper limit deviation of the shaft diameter, mm | 0.015 | 0.015
Lower limit deviation of the shaft diameter, mm | 0.040 | 0.040
Shaft hole diameter (in bearing zone), mm | 0 | 0
Housing bore nominal diameter, mm | 215 | 175
Upper limit deviation of the housing bore diameter, mm | 0.000 | 0.000
Lower limit deviation of the housing bore diameter, mm | 0.046 | 0.040
Housing outer diameter, mm | 260 | 210

Table 5. Solution vector

<table>
<thead>
<tr>
<th>$\delta_x$ [(\mu m)]</th>
<th>Bearing 1</th>
<th>Bearing 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_y$ ($\delta_1$)</td>
<td>$\delta_y$ ($\delta_1$)</td>
<td>$\theta_x$ ($\theta_H$)</td>
</tr>
<tr>
<td>$\delta_z$ ($\delta_2$)</td>
<td>$\delta_z$ ($\delta_2$)</td>
<td>$\delta_z$ ($\delta_2$)</td>
</tr>
<tr>
<td>$\theta_y$ ($\theta_H$)</td>
<td>$\theta_y$ ($\theta_H$)</td>
<td>$\theta_z$ ($\theta_V$)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\delta_x$</th>
<th>$\delta_y$</th>
<th>$\delta_z$</th>
<th>$\theta_x$</th>
<th>$\theta_y$</th>
<th>$\theta_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.17</td>
<td>5.23</td>
<td>-12.39</td>
<td>-98.56</td>
<td>-2.79</td>
<td>-1.72</td>
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Table 6. Bearing loads and moments (sign according to own Cartesian system of coordinates)

<table>
<thead>
<tr>
<th>Bearing</th>
<th>Axial load</th>
<th>Radial loads</th>
<th>Moments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$F_x$ [kN]</td>
<td>$F_y$ [kN]</td>
<td>$F_z$ [kN]</td>
</tr>
<tr>
<td>Bearing 1</td>
<td>-2.265</td>
<td>-14.841</td>
<td>-6.931</td>
</tr>
<tr>
<td>Bearing 2</td>
<td>-0.005</td>
<td>5.924</td>
<td>-1.540</td>
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</table>

Table 7. Lubricant and lubrication system type and cleanliness

<table>
<thead>
<tr>
<th>Lubrication characteristic</th>
<th>Bearing 1</th>
<th>Bearing 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kinematic viscosity of oil at 40 °C, mm²/s</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Viscosity index</td>
<td>98</td>
<td>98</td>
</tr>
<tr>
<td>EP additives</td>
<td>NO</td>
<td>NO</td>
</tr>
<tr>
<td>Lubrication type</td>
<td>Oil lubrication without filtration</td>
<td>Oil lubrication without filtration</td>
</tr>
<tr>
<td>Cleanliness codes (ISO 4406, 1999)</td>
<td>-/16/13</td>
<td>-/16/13</td>
</tr>
<tr>
<td>Reference kinematic viscosity of oil, mm²/s</td>
<td>10.39</td>
<td>14.65</td>
</tr>
<tr>
<td>Kinematic viscosity of oil at working temperature, mm²/s</td>
<td>14.65</td>
<td>14.65</td>
</tr>
<tr>
<td>Viscosity ratio</td>
<td>1.41</td>
<td>1.29</td>
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Table 8. Bearing lives

<table>
<thead>
<tr>
<th>Standard</th>
<th>Bearing</th>
<th>Dynamic equivalent load $P_r$ [kN]</th>
<th>Basic rating life $L_{10h}$ [hours]</th>
<th>Factor $a_{ISO}$</th>
<th>Modified rating life $L_{10mah}$ [hours]</th>
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<tbody>
<tr>
<td>ISO 16281:2008</td>
<td>Bearing 1</td>
<td>19.404</td>
<td>5012</td>
<td>7.40</td>
<td>37104</td>
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<td></td>
<td>Bearing 2</td>
<td>10.845</td>
<td>662</td>
<td>1.68</td>
<td>1113</td>
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<td>ISO 281:2007</td>
<td>Bearing 1</td>
<td>18.889</td>
<td>5434</td>
<td>7.78</td>
<td>42270</td>
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<td>Bearing 2</td>
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<td>1916</td>
<td>2.57</td>
<td>4918</td>
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</tbody>
</table>
The calculation results are listed in Table 8. As it can be seen, the results fundamentally differ. The reason is the noticeable distinction between the dynamic equivalent loads taken into account during the calculation. Mentioning only bearing 2 (the most loaded one), now the load ratio is $P_{rISO281}/P_{rISO16281}=0.7$, that is less than 1. Accordingly, the life ratios are $L_{10ISO281}/L_{10ISO16281}=2.9$ and $L_{10mdISO281}/L_{10mdISO16281}=4.4$.

5 CONCLUDING REMARKS

It has been demonstrated that the proposed method ensures a rapid, complete, and accurate bearing life calculation according to ISO 16281 even for double-row bearings. The approach has gained its irrefragable reliability through the use of the slope-deflection method, which facilitated an easy connection of the shaft behavior under loads to any bearing dynamic model. Note that the data preparations, the setting and computing times are extremely low and the accuracy of the method is more than acceptable.

This approach is therefore recommended for bearing life calculation in early and middle stages of a certain project and can become a valuable companion to the most sophisticated tools which are supposed to be used mainly to refine and validate the design solution obtained up to that point. Moreover, the simplicity and the rapidity of the proposed tool makes it suitable to be used within an optimization program (where bearing lives must be calculated several millions of times) which will have the ultimate goal to bring the design solution as close as possible to the ideal one.

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7 REFERENCES